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Parametric study of irreversible Stirling and Ericsson cryogenic refrigeration cycles

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Abstract

This communication presents a parametric study of irreversible Stirling and Ericsson cryogenic refrigerator cycles including external and internal irreversibilities along with finite heat capacities of external reservoirs. The external irreversibility is due to finite temperature difference between the working fluid and external fluids in the source/sink reservoirs, while the internal irreversibilities are due to regenerative heat loss and other entropy generations in the cycle. The cooling load is maximized for the given power input. The cooling coefficient of performance, the heat transfers to and from the refrigerator and the working fluid temperatures at these conditions have been evaluated. The effect of different parameters (reservoirs fluid temperature, the effectivenesses, heat capacitance rates of the external reservoirs and irreversibility parameter) on the performance of these cycles has been studied. It is found that the effect of the internal irreversibility parameter is more pronounced than that of other parameters. © 2002 Published by Elsevier Science Ltd.

Keywords: Internal irreversibility parameter; Stirling and Ericsson cryogenic refrigeration cycles; Optimum performance

1. Introduction

Stirling and Ericsson cycles are two of the important cycle models of cryogenic refrigeration systems. These cycles have been utilized by a number of engineering firms in the construction of practical systems for the production of very low temperatures. This has promoted the

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Nomenclature

A	area (m^2)
C	heat capacitance (kW K^{-1})
C_f	specific heat ($\text{kJ kg}^{-1} \text{K}^{-1}$)
COP	coefficient of performance
P	power (kW)
p	pressure (k Pa)
Q	heat (kJ)
R	cooling load (kW)/gas constant ($\text{kJ kg}^{-1} \text{K}^{-1}$)
T	temperature (K)
t	time (s)
U	overall heat transfer coefficient ($\text{kW m}^{-2} \text{K}^{-1}$)
v	volume (m^3)

Greek

ε	effectiveness
λ_1/λ_2	pressure/volume ratio

Subscript

c	cold side/sink side
h, H	hot side/heat source
L	heat sink/cooling
m	optimum operating condition
R	regenerator
s	isentropic/ideal
p	at constant pressure
v	at constant volume
1, 2, 3, 4	state points

development of new designs of these cycles. The concept of finite time thermodynamics was introduced by Curzon–Ahlborn [1] with a novel work on the Carnot heat engine in which they remarked that the efficiency of an engine operating at maximum power is given by ($\eta_m = 1 - \sqrt{T_L/T_H}$), which is always smaller than the well known Carnot efficiency ($\eta_C = 1 - T_L/T_H$) and agrees much better with the measured efficiencies of operating installations. Wu [3] also applied the similar concept on the finite time Carnot heat engine with finite heat capacity of external reservoirs. It is also desirable to have the corresponding results for the coefficient of performance (COP) of cryogenic refrigeration. Leff and Teeters [2] have noted that the straight forward C – A calculations will not work for a reversed Carnot cycle because there is no “natural maximum” in these cycles. Blanchard [4] has applied the Lagrangian method of undetermined multipliers to find the COP of an endoreversible Carnot heat pump operated at minimum power input for a given heating load. In recent years, Wu [5,6], Wu and co-workers [7–10,12], Chen [11], Kaushik [13], Kaushik and Kumar [14], Kumar [15] and Kaushik et al. [16] have also studied the simple Stirling

refrigerator, Stirling and Ericsson heat pumps and other cycles using finite time thermodynamics and presented the optimum performance parameters as a function of operating conditions.

In this paper, we have presented a more general analysis of cryogenic Stirling and Ericsson refrigerators with both external and internal irreversibilities arising due to the finite heat capacitance, finite temperature differences between the cycles and the source/sink reservoirs and regenerative losses and the entropy generations within the cycles. The effects of operating inlet temperatures and heat capacitance rates of the source/sink reservoirs, the effectivenesses of the various heat exchangers and the internal irreversibility parameter on the heat transfers, the power input, the cooling load, cooling COP and the working fluid temperatures of the Ericsson and Stirling refrigerator cycles have been studied.

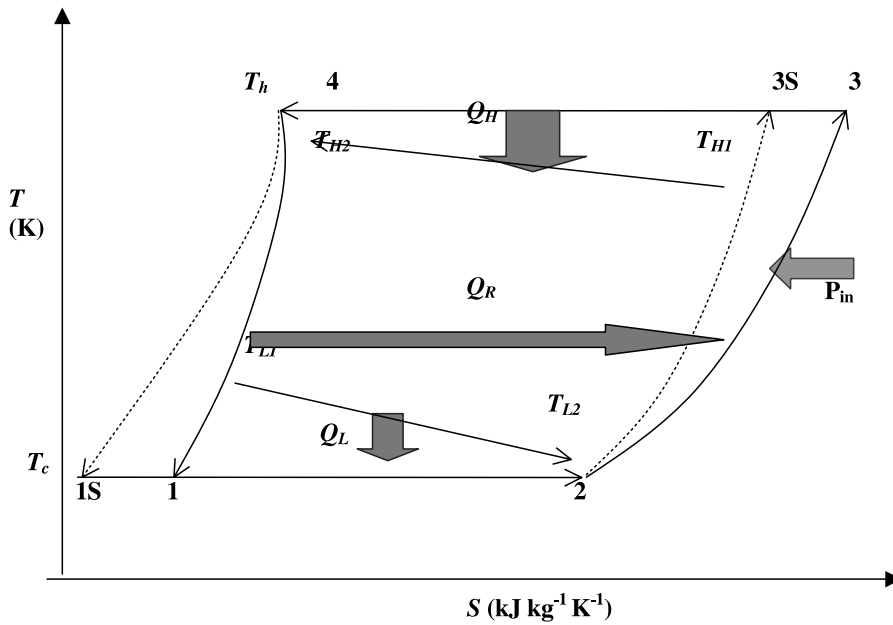
2. System description

It is well known that the working substance of Ericsson and Stirling cycles may be a gas, and the Stirling cycle consists of two isothermal and two isochoric processes, while the Ericsson cycle consists of two isothermal and two isobaric processes, as shown in the T – S and schematic diagrams in Figs. 1 and 2. These cycles approximate the expansion stroke of a real cycle as an isothermal process 1–2 with irreversible heat addition (at temperature T_c from a heat source of finite heat capacity whose temperature varies from T_{L1} to T_{L2}). The heat addition to the working fluid from the regenerator is modeled as isobaric (in the Ericsson cycle) or isochoric (in the Stirling cycle) processes 2–3 and 2–3S in real and ideal cycles, respectively. The compression stroke is modeled as an isothermal process 3–4 with irreversible heat rejection (at temperature T_h) to the heat sink of finite heat capacity whose temperature varies from T_{H1} to T_{H2} . Finally, the heat rejection to the regenerator is modeled as isobaric/isochoric processes 4–1 and 4–1S in real and ideal Ericsson/Stirling refrigerators, respectively, thereby completing the cycle.

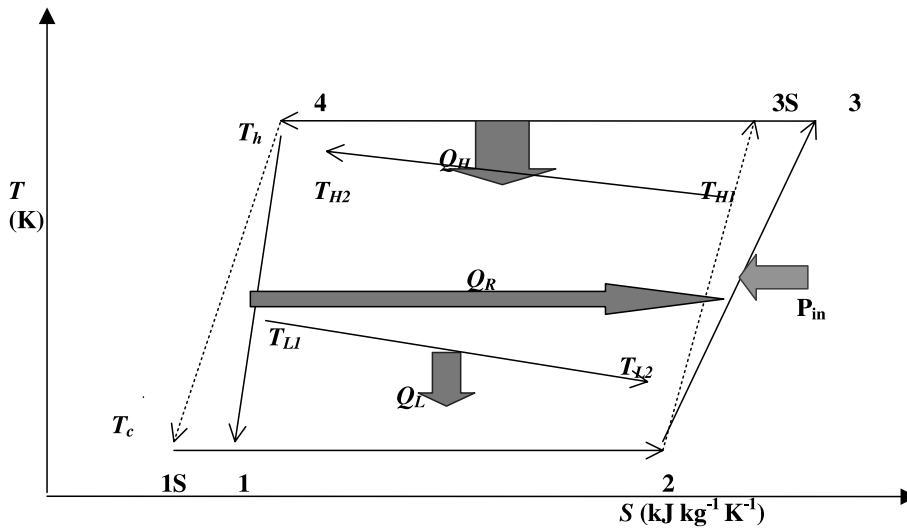
As mentioned earlier, the heat transfer processes 1–2 and 3–4 in real cycles must occur in finite time. This requires that these heat transfer processes must proceed through a finite temperature difference and, therefore, are defined as being externally irreversible. Also, the entropy change during process 3–4 is more than the entropy change during process 1–2. Thus, there is some net entropy generation per cycle, and the ratio of the entropy change in heat addition (expansion) to that of heat rejection (compression) is introduced as an internal irreversibility parameter ($R_{\Delta S}$), which is always less than unity and also affects the performance of these cycles. There is also some heat loss through the regenerator, as an ideal regeneration requires infinite regeneration time or area, which is not the case in practice. Hence, it is desirable to consider a real regenerator. By considering all these factors, these refrigerator models become irreversible. The external irreversibility is due to the finite temperature difference between the cycles and the external reservoirs, while the internal irreversibilities are due to the regenerative loss and entropy generation within the cycles.

3. Thermodynamic analysis

Let Q_c be the amount of heat absorbed from the source at temperature T_c of the cycle and Q_h be the amount of heat released to the sink at temperature T_h of the cycle, then:



(a): Stirling Refrigerator Cycle



(b): Ericsson Refrigerator Cycle

Fig. 1. T - S diagrams of irreversible Stirling and Ericsson refrigeration cycles.

$$Q_h = T_h(S_3 - S_4) = C_H(T_{H2} - T_{H1})t_H \quad (1)$$

$$Q_c = T_c(S_2 - S_1) = C_L(T_{L1} - T_{L2})t_L \quad (2)$$

where $(S_2 - S_1) = nR \ln \lambda_1$ and $(S_3 - S_4) = nR \ln \lambda_2$, λ_1 and λ_2 are the volume (for the Stirling cycle) and pressure (for the Ericsson cycle) ratios for the two regenerative processes, n is the number of

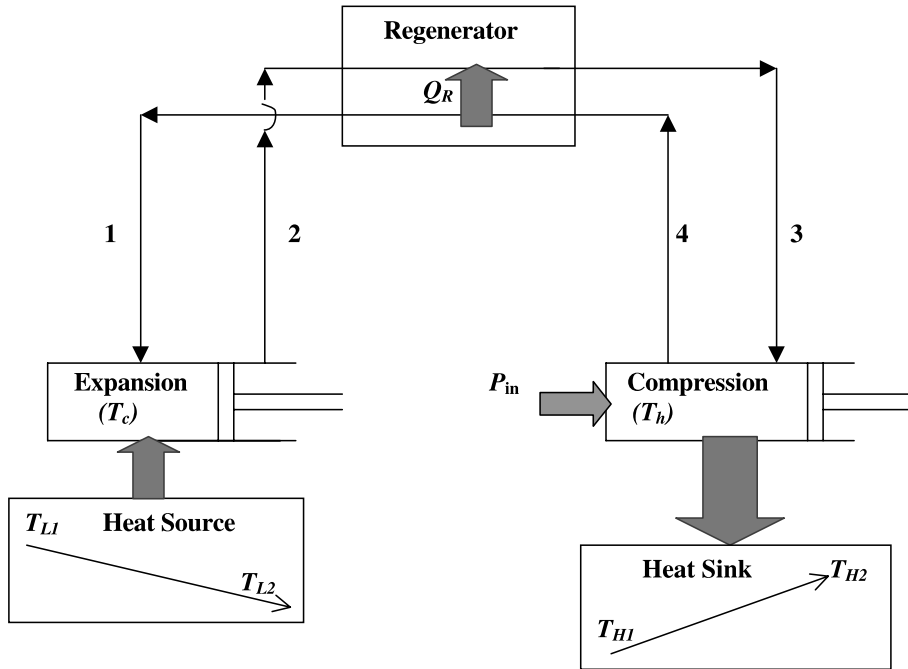


Fig. 2. Schematic of irreversible Stirling/Ericsson refrigeration cycle.

moles of the working fluid, R is the universal gas constant and C_H , C_L and t_H , t_L are the heat capacitance rates of the source/sink reservoirs and heat addition/rejection times, respectively. Also, from heat transfer theory, the amounts of heat Q_h and Q_c will be proportional to the log mean temperature difference (LMTD), i.e.

$$Q_h = U_H A_H (\text{LMTD})_H t_H \quad (3)$$

$$Q_c = U_L A_L (\text{LMTD})_L t_L \quad (4)$$

where $U_H A_H$ and $U_L A_L$ are the overall heat transfer coefficient-area products and $(\text{LMTD})_H$ and $(\text{LMTD})_L$ are the log mean temperature differences on the sink and source sides, respectively, defined as

$$(\text{LMTD})_H = \frac{(T_h - T_{H1}) - (T_h - T_{H2})}{\left(\ln \frac{(T_h - T_{H1})}{(T_h - T_{H2})} \right)} \quad (5)$$

$$(\text{LMTD})_L = \frac{(T_{L1} - T_c) - (T_{L2} - T_c)}{\left(\ln \frac{(T_{L1} - T_c)}{(T_{L2} - T_c)} \right)} \quad (6)$$

As these cycles, in general, do not possess the condition of perfect regeneration, it is desirable to consider the regenerator loss, which is proportional to the temperature difference between the two isothermal processes as assumed by earlier workers [14–16], i.e.

$$\Delta Q_R = nC_f(1 - \varepsilon_R)(T_h - T_c) \quad (7)$$

where C_f is the specific heat of the working fluid ($C_f = C_v$ for the Stirling cycle and $C_f = C_p$ for the Ericsson cycle) and ε_R is the effectiveness of the regenerator.

Owing to the influence of irreversibility of finite heat transfer, the regenerative time should be finite, similar to the two isothermal processes [12–16], and will be

$$t_R = t_3 + t_4 = 2\alpha(T_h - T_c) \quad (8)$$

where α is the proportionality constant, which is independent of the temperature difference but depends on the properties of the regenerative material. Thus, the total cycle time t_{cycle} will be:

$$t_{\text{cycle}} = (t_H + t_L + t_R) \quad (9)$$

When all the irreversibilities mentioned above are taken into account, the net amount of heat released to the sink and absorbed from the source are given by

$$Q_H = Q_h - \Delta Q_R \quad (10)$$

$$Q_L = Q_c - \Delta Q_R \quad (11)$$

It is well known that the power input, cooling load and the COP are the important parameters of any refrigerator. Using the above equations, we obtain expressions for the power input, cooling load and cooling COP as follows:

$$P = \frac{(Q_H - Q_L)}{t_{\text{cycle}}} = \frac{(Q_h - Q_c)}{(t_H + t_L + t_R)} \quad (12)$$

$$R_L = \frac{Q_L}{t_{\text{cycle}}} = \frac{Q_L}{(t_H + t_L + t_R)} \quad (13)$$

$$\text{COP}_L = \frac{R_L}{P} = \frac{Q_L}{(Q_h - Q_c)} \quad (14)$$

The second law of thermodynamics for an irreversible cycle gives

$$\frac{Q_c}{T_c} - \frac{Q_h}{T_h} < 0 \quad \text{or} \quad \frac{Q_c}{T_c} = R_{\Delta S} \frac{Q_h}{T_h} \quad (15)$$

where $R_{\Delta S}$ is the irreversibility parameter and is less than unity for a real cycle. Thus, from Eqs. (10)–(15), we have:

$$P = \frac{(x - R_{\Delta S})}{\left[\frac{x}{k_1(xy - T_{H1})} + \frac{R_{\Delta S}}{k_2(T_{L1} - y)} + b_1(x - 1) \right]} \quad (16a)$$

$$R_L = \frac{[R_{\Delta S} - a_1(x - 1)]}{\left[\frac{x}{k_1(xy - T_{H1})} + \frac{R_{\Delta S}}{k_2(T_{L1} - y)} + b_1(x - 1) \right]} \quad (16b)$$

$$\text{COP}_L = \frac{[R_{\Delta S} - a_1(x - 1)]}{(x - R_{\Delta S})} \quad (16c)$$

where $k_1 = \varepsilon_H C_H$, $k_2 = \varepsilon_L C_L$, $x = T_h T_c$, $y = T_c$, $b_1 = 2\alpha/(nR \ln \lambda_1)$ and $a_1 = C_f(1 - \varepsilon_R)/R \ln \lambda_1$.

The purpose of any refrigerator is to extract as much heat as possible from the source (space to be cooled) with the expenditure of as little work as possible. This implies that we should do our best to minimize the power input for a given cooling load or maximize the cooling load for a given power input. To do so, we introduce the Lagrangian

$$L = R_L + \lambda P = \frac{[R_{\Delta S} - a_1(x - 1)] + \lambda(x - R_{\Delta S})}{\left[\frac{x}{k_1(xy - T_{H1})} + \frac{R_{\Delta S}}{k_2(T_{L1} - y)} + b_1(x - 1)\right]} \quad (17)$$

where λ is the Lagrangian multiplier. From the Euler–Lagrange equation

$$\frac{\partial L}{\partial y} = 0 \quad (18)$$

and with Eq. (18), we find the optimal relation:

$$x(T_{L1} - y) = \sqrt{\frac{k_1 R_{\Delta S}}{k_2}}(xy - T_{H1}) \quad (19)$$

Substituting Eq. (19) into Eqs. (16a)–(16c) gives:

$$P = \frac{(x - R_{\Delta S})}{\left[\frac{xk_4}{(xT_{L1} - T_{H1})} + b_1(x - 1)\right]} \quad (20a)$$

$$R_L = \frac{[R_{\Delta S} - a_1(x - 1)]}{\left[\frac{xk_4}{(xT_{L1} - T_{H1})} + b_1(x - 1)\right]} \quad (20b)$$

$$\text{COP}_L = \frac{[R_{\Delta S} - a_1(x - 1)]}{(x - R_{\Delta S})} \quad (20c)$$

where

$$k_3 = \sqrt{\frac{k_1 R_{\Delta S}}{k_2}} \quad k_4 = (1 + k_3) \left[\frac{1}{k_1} + \frac{R_{\Delta S}}{k_2 k_3} \right] \quad (21)$$

Eq. (20b) shows that the cooling load is not a monotonically increasing function of power input, and there exists a maximum value of cooling load. Thus, maximizing R_L with respect to x yields

$$x_{\text{opt}} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \quad (22)$$

where $A = k_4 a_1 T_{L1} + b_1 R_{\Delta S} T_{L1}^2$; $B = -2R_{\Delta S} b_1 T_{H1} T_{L1}$ and $C = -k_4(a_1 + R_{\Delta S})T_{H1} + b_1 T_{H1}^2$.

Substituting Eq. (22) into Eqs. (20a)–(20c), we find the optimal values of the power input, cooling load and cooling COP of these refrigerators.

4. Special cases

1. When $\varepsilon_R = 1.00$, these cycles possess the condition of perfect regeneration. The performance of these obtained cycles is also given by Eqs. (20a)–(20c) by substituting $a_1 = 0.0$. Thus, the performance characteristics of the Ericsson and Stirling refrigerator cycles with perfect regeneration are equivalent to that of similar Carnot refrigerators operating at the same condition in which the time of the two adiabatic processes is given by Eq. (8), although physically, for finite regeneration time, ε_R should be less than unity. This shows that in the investigation of these cycles, it would be impossible to obtain new conclusions if the regenerative losses were not considered.
2. When $t_R = \gamma(t_H + t_L)$ i.e. when the time of regeneration is proportional to that of the two isothermal processes. The performance characteristics are given by

$$P = \frac{(x_1 - R_{\Delta S})(x_1 T_{L1} - T_{H1})}{(1 + \gamma)x_1 k_4} \quad (23a)$$

$$R_L = \frac{[R_{\Delta S} - a_1(x_1 - 1)](x_1 T_{L1} - T_{H1})}{(1 + \gamma)x_1 k_4} \quad (23b)$$

$$\text{COP}_L = \frac{[R_{\Delta S} - a_1(x_1 - 1)]}{(x_1 - R_{\Delta S})} \quad (23c)$$

where

$$x_1 = \frac{-B_1 \pm \sqrt{B_1^2 - 4A_1C_1}}{2A_1} \quad (24)$$

and $A_1 = a_1 T_{L1}$, $B_1 = -2a_1 T_{H1}$ and $C_1 = (a_1 + R_{\Delta S})T_{H1}$.

5. Discussion of results

In order to have a numerical appreciation of the results for the cryogenic refrigerators, we have considered the heat sink and heat source temperatures $T_{H1} = 290\text{--}325\text{ K}$ and $T_{L1} = 180\text{--}260\text{ K}$, respectively, the effectivenesses of the heat exchangers (ε_H , ε_L and ε_R) in the range 0.60–1.00, the pressure/volume ratios ($p_1/p_2 = v_2/v_1$) = 2.50 and the irreversibility parameter $R_{\Delta S}$ in the range 0.50–1.00. We have studied the effect of each of these parameters (while keeping the others constant) on the power input, the cooling load and the cooling COP of both refrigerators, and the results are given below.

Table 1(panels A–C) show the effects of the various heat exchangers (ε_H , ε_L and ε_R) on the heat transfer to and from the refrigerators, the regenerative heat transfer, the power input, the cooling load, the cooling COP and the working fluid temperatures.

Effect of ε_H : It can be seen from Table 1(panel A) that as the effectiveness of the sink side heat exchanger increases, the heat transfer to the refrigerators, the power input, the cooling load and the cooling COP increase, while the heat transfer from the refrigerators, the regenerative heat transfer and the working fluid temperatures decrease. The effect of ε_H is more pronounced for the cooling load and less pronounced for the cold side working fluid temperature.

Table 1

Effect of ε_H (panel A, $\varepsilon_L = 0.75$, $\varepsilon_R = 0.90$), ε_L (panel B, $\varepsilon_H = 0.75$, $\varepsilon_R = 0.90$) and ε_R (panel C, $\varepsilon_L = \varepsilon_H = 0.75$) on the performance of Stirling/Ericsson refrigerators ($T_{L1} = 250$ K, $T_{H1} = 300$ K, $C_H = C_L = 1.0$ and $R_{\Delta S} = 0.80$)

	Q_H	Q_L	Q_R	P_m	R_L	COP_L	T_h	T_c
ε_H (panel A)								
0.50	77.81	57.91	48.79	0.90	0.83	0.92	316.50	240.83
0.60	77.56	57.95	48.10	0.92	0.87	0.94	315.27	240.68
0.70	77.37	57.98	47.55	0.95	0.90	0.95	314.29	240.55
0.75	77.28	57.99	47.31	0.96	0.92	0.96	313.87	240.50
0.80	77.20	58.00	47.09	0.97	0.93	0.96	313.48	240.45
0.85	77.13	58.01	46.90	0.97	0.94	0.97	313.13	240.40
0.90	77.07	58.02	46.71	0.98	0.96	0.97	312.80	240.36
0.95	77.01	58.03	46.54	0.99	0.97	0.98	312.50	240.32
1.00	76.95	58.04	46.38	1.00	0.98	0.98	312.22	240.29
ε_L (panel B)								
0.50	77.09	57.42	48.22	0.90	0.84	0.93	313.51	238.73
0.60	77.18	57.69	47.80	0.93	0.87	0.94	313.68	239.55
0.70	77.25	57.90	47.46	0.95	0.90	0.95	313.81	240.21
0.75	77.28	57.99	47.31	0.96	0.92	0.96	313.87	240.50
0.80	77.31	58.07	47.18	0.97	0.93	0.96	313.92	240.76
0.85	77.34	58.15	47.05	0.97	0.94	0.97	313.96	241.00
0.90	77.36	58.22	46.94	0.98	0.95	0.97	314.01	241.22
0.95	77.38	58.28	46.83	0.99	0.96	0.98	314.05	241.42
1.00	77.40	58.34	46.73	0.99	0.97	0.98	314.08	241.61
ε_R (panel C)								
0.70	67.78	53.13	27.95	0.37	0.09	0.24	303.31	247.58
0.75	69.68	54.01	32.02	0.56	0.22	0.39	305.58	245.98
0.80	71.87	55.08	36.59	0.72	0.40	0.56	308.10	244.26
0.85	74.39	56.39	41.68	0.85	0.64	0.75	310.87	242.42
0.90	77.28	57.99	47.31	0.96	0.92	0.96	313.87	240.50
0.95	80.57	59.91	53.49	1.04	1.23	1.19	317.09	238.51
1.00	83.56	62.61	57.08	1.16	1.74	1.50	317.76	238.09

Effect of ε_L : Table 1(panel B) shows that as effectiveness of the source side heat exchanger increases, the heat transfer to and from the refrigerators, the power input, the cooling load, the cooling COP and the working fluid temperatures increase, while the regenerative heat transfer decreases. The effect of ε_L is more pronounced for the heating load and less pronounced for the sink side working fluid temperature.

Effect of ε_R : Table 1(panel C) shows that as the effectiveness of the regenerator increases, the heat transfer to and from the refrigerators, the regenerative heat transfer, the power input, the cooling load, the cooling COP and the sink side working fluid temperature increase, while the source side working fluid temperature decreases. The effect of ε_R is more pronounced for the cooling load and less pronounced for the source side working fluid temperature.

Table 2(panels A and B) show the effects of the source and sink inlet temperatures (T_{L1} and T_{H1}) on the heat transfers, the power input, the cooling load, the cooling COP and the working fluid temperatures of these cycles.

Table 2

Effect of T_{L1} (panel A, $T_{H1} = 300$ K), T_{H1} (panel B, $T_{L1} = 250$ K) on the performance of Stirling/Ericsson refrigerators ($\varepsilon_H = \varepsilon_L = 0.75$, $\varepsilon_R = 0.90$, $C_H = C_L = 1.0$ and $R_{\Delta S} = 0.80$)

	Q_H	Q_L	Q_R	P_m	R_L	COP_L	T_h	T_c
<i>T_{L1} (panel A)</i>								
180	70.87	37.81	81.06	0.50	0.04	0.08	303.74	178.04
190	71.94	40.53	77.03	0.65	0.11	0.16	306.12	186.66
200	72.93	43.34	72.56	0.74	0.19	0.26	307.99	195.47
210	73.86	46.21	67.81	0.80	0.30	0.37	309.53	204.37
220	74.76	49.12	62.87	0.85	0.42	0.50	310.84	213.35
230	75.62	52.06	57.78	0.89	0.56	0.63	311.97	222.37
240	76.46	55.01	52.59	0.92	0.73	0.79	312.97	231.42
250	77.28	57.99	47.31	0.96	0.92	0.96	313.87	240.50
260	78.09	60.98	41.96	0.99	1.14	1.15	314.67	249.60
<i>T_{H1} (panel B)</i>								
290	75.40	58.49	41.47	0.98	1.09	1.12	304.26	239.94
295	76.34	58.24	44.40	0.97	1.00	1.03	309.07	240.22
300	77.28	57.99	47.31	0.96	0.92	0.96	313.87	240.50
305	78.22	57.74	50.22	0.95	0.84	0.89	318.65	240.77
310	79.15	57.49	53.12	0.94	0.77	0.82	323.43	241.05
315	80.08	57.24	56.02	0.93	0.71	0.76	328.19	241.32
320	81.01	56.99	58.91	0.92	0.65	0.71	332.95	241.60
325	81.94	56.74	61.79	0.91	0.60	0.65	337.69	241.87

Effect of T_{H1} : It can be seen from Table 2(panel B) that as the inlet temperature of the sink fluid increases, the heat transfer from the refrigerators, the regenerative heat transfer and the working fluid temperatures increase, while the heat transfer to the refrigerators, the power input, the cooling load and the cooling COP decrease. The effect of T_{H1} is more pronounced for the cooling load and less pronounced for the source side working fluid temperature. Thus, it is desirable to have lower T_{H1} for better performance of both refrigerators.

Effect of T_{L1} : Table 2(panel A) shows that as T_{L1} increases, the heat transfer to and from the refrigerators, the power input, the cooling load, the cooling COP and the working fluid temperatures increase, while the regenerative heat transfer decreases. The effect of T_{L1} is more pronounced for the cooling load and less pronounced for the sink side working fluid temperature.

Table 3(panels A–C) show the effects of the heat capacitance rates (C_H and C_L) and the internal irreversibility parameter ($R_{\Delta S}$) on the heat transfers, the power input, the cooling load, the cooling COP and the working fluid temperatures of these refrigerators.

Effect of C_H : Table 3(panel A) shows that as the heat capacitance rates of the sink side fluid increases, the heat transfers (Q_H and Q_R) and the working fluid temperatures decrease, but the heat transfer to the refrigerators, the power input, the cooling load and the cooling COP increase. The effect of C_H is more pronounced for the cooling load and less pronounced for the heat transfer to the refrigerators.

Effect of C_L : Table 3(panel B) shows that as the heat capacitance rate of the source side fluid increases, the heat transfers to and from the refrigerators (Q_H and Q_L), the power input, cooling load, the cooling COP and the working fluid temperatures increase, while the regenerative heat

Table 3

Effect of C_H (panel A, $C_L = 1.0$, $R_{\Delta S} = 0.80$), C_L (panel B, $C_H = 1.0$, $R_{\Delta S} = 0.80$) and $R_{\Delta S}$ (panel C, $C_H = C_L = 1.0$) on the performance of Stirling/Ericsson refrigerators ($\varepsilon_H = \varepsilon_L = 0.75$, $\varepsilon_R = 0.90$, $T_{L1} = 250$ K, $T_{H1} = 300$ K)

	Q_H	Q_L	Q_R	P_m	R_L	COP_L	T_h	T_c
<i>C_H (panel A)</i>								
0.50	78.23	57.85	49.97	0.85	0.76	0.90	318.59	241.10
0.60	77.96	57.89	49.21	0.88	0.80	0.91	317.24	240.93
0.70	77.74	57.92	48.60	0.90	0.84	0.93	316.16	240.79
0.80	77.56	57.95	48.10	0.92	0.87	0.94	315.27	240.68
0.90	77.41	57.97	47.68	0.94	0.89	0.95	314.51	240.58
1.00	77.28	57.99	47.31	0.96	0.92	0.96	313.87	240.50
1.10	77.17	58.00	46.99	0.97	0.94	0.97	313.30	240.43
1.20	77.07	58.02	46.71	0.98	0.96	0.97	312.80	240.36
1.30	76.98	58.03	46.46	0.99	0.97	0.98	312.36	240.31
1.40	76.90	58.04	46.24	1.00	0.99	0.98	311.96	240.26
1.50	76.83	58.05	46.03	1.01	1.00	0.99	311.59	240.21
<i>C_L (panel B)</i>								
0.50	76.93	56.97	48.94	0.86	0.77	0.90	313.22	237.33
0.60	77.03	57.26	48.48	0.89	0.81	0.92	313.41	238.23
0.70	77.11	57.49	48.11	0.91	0.85	0.93	313.56	238.96
0.80	77.18	57.69	47.80	0.93	0.87	0.94	313.68	239.55
0.90	77.23	57.85	47.54	0.94	0.90	0.95	313.78	240.06
1.00	77.28	57.99	47.31	0.96	0.92	0.96	313.87	240.50
1.10	77.32	58.11	47.11	0.97	0.94	0.97	313.94	240.88
1.20	77.36	58.22	46.94	0.98	0.95	0.97	314.01	241.22
1.30	77.39	58.31	46.78	0.99	0.97	0.98	314.06	241.52
1.40	77.42	58.40	46.64	1.00	0.98	0.98	314.12	241.79
1.50	77.45	58.48	46.51	1.01	0.99	0.99	314.16	242.03
<i>R_{ΔS} (panel C)</i>								
0.50	76.70	59.79	41.46	1.07	0.32	0.30	309.17	244.87
0.60	76.93	59.16	43.56	1.08	0.50	0.46	310.92	243.38
0.70	77.12	58.56	45.49	1.04	0.70	0.67	312.47	241.92
0.75	77.20	58.27	46.42	1.00	0.80	0.80	313.18	241.21
0.80	77.28	57.99	47.31	0.96	0.92	0.96	313.87	240.50
0.85	77.36	57.71	48.19	0.90	1.03	1.14	314.52	239.79
0.90	77.43	57.43	49.04	0.84	1.15	1.36	315.15	239.10
0.95	77.51	57.15	51.01	0.78	1.28	1.64	315.78	238.41
1.00	78.15	55.27	53.62	0.72	1.39	1.92	320.47	335.29

transfer (Q_R) decreases. The effect of C_L is more pronounced for the cooling load and less pronounced for the sink side working fluid temperature.

Since the higher value of C_H decreased the heat transfers and increased the performance, while the higher value of C_L increased the heat transfers but slightly decreased the performance, it is desirable to have a higher C_H rather than a higher C_L .

Effect of $R_{\Delta S}$: Table 3(panel C) shows that as the internal irreversibility parameter increases, the heat transfers (Q_H and Q_R), the cooling load, the cooling COP and the sink side working fluid temperature increase, while the heat transfer to the refrigerators, the power input and the source

side working fluid temperature decrease. The effect of $R_{\Delta S}$ is more pronounced for the power input and less pronounced for the sink side working fluid temperature. Since the effect of $R_{\Delta S}$ is more pronounced than the other parameters on the performance of both refrigerators, it is desirable to have higher $R_{\Delta S}$ for better performance of both cycles.

6. Conclusions

The performance characteristics of irreversible Ericsson and Stirling cryogenic refrigerator cycles have been evaluated, including external as well as internal irreversibilities. The external irreversibility is due to the finite temperature differences and the internal irreversibilities are due to the regenerative loss and entropy generation in the cycle. It is found that the effectiveness of each heat exchanger, the heat capacitance rates on the sink side fluid and the irreversibility parameters should be higher, while the sink side inlet temperature of the fluids and the heat capacitance rates on the source side should be lower for better performance of both cycles. Since the larger heat capacitance allows the fluid to reject heat at a lower temperature, it is desirable to have a higher capacitance rate on the sink side in comparison to that on the source side (i.e. $C_H > C_L$) for better performance of both refrigerators. Hence, the present analysis will be useful and more general for evaluating the performance of these cycles and other regenerative cycles as well.

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